THE USE OF THERMAL NETWORKS METHOD FOR THE MEASUREMENT OF THERMAL DIFFUSIVITY OF POROUS MATERIALS USED IN BUILDINGS

A. EL BOUARDI[1], F. AGHBALOU[2] AND F. BADIA[2]
Thermal, Solar Energy and Environment Laboratory. Abdelmalek Essaadi University.
Tetouan, Morocco
Phone 212 39 97 24 23 / Fax 212 39 99 45 00, E-mail: abouardi@fst.ac.ma
[2] Centro de investigación en energía aplicada (CREA). C/ Jaume II, 69, Lleida, Spain
Tel +34 973 70 27 27, Fax +34 973 70 27 02 fouad@diei.UdL.es, fbadia@macs.udl.es,

ABSTRACT :
Thermal diffusivity is an essential parameter which allows for a better understanding of a thermal behaviour of dwelling in a dynamic regime. It's therefore so important to define the aptitude that a wall has to release energy more or less fast to the surrounding we want to control. This faculty is related to a more significant definition of thermal diffusivity of different granular or compact material constituting the wall. This paper presents a thermal diffusivity method based on a thermal networks and iterative square least method.

KEY WORDS: Thermal network, Measurements, modelling, identification, thermal diffusivity, granular materials, compact materials

NOMENCLATURE

a	Thermal diffusivity (m^2/s)
Bi	Biot number
c	Specific heat (J/kg.°K)
e	Thickness (m)
h	Heat transfer coefficient (W/m °C)
p	Laplace variable
q	flux density (W/m^2)
S	Sample surface (m^2)
T	Temperature (°C)
t	Time (s)
t_o	Irradiation period (s)
λ	Thermal conductivity (w/m °C)
ρ	Apparent density (kg/m^3)
$\overline{T}$	Temperature Laplace transform
ϕ	Flux Laplace transform

Indices
c : calculated
exp: experimental
0 : at $x = 0$
e : at $x = e$
s : system
l : metallic plates

1. THERMAL MODEL SOLUTION

The method used to solve the heat transfer equation within the ’’ Sandwich ’’, Fig.1. a, is the so called thermal networks. Assumptions are:

- The ″ Sandwich ″ is composed of two identical plates of thermal diffusivity $a_1 = \frac{\lambda_1}{\rho_1 c_1}$ and granular medium of thermal diffusivity $a = \frac{\lambda}{\rho c}$.
- Contact thermal resistance between granular medium and plates are neglected. This assumption is justified for slim granular medium
- The heat transfer is unidirectional
- Wall lateral losses are neglected
- Initially, the system is considered to be in thermal equilibrium: no internal source

According to the equivalent electrical scheme (Fig. 1.b), the linear relation that relates input parameters $(\overline{T_0}, \phi_0)$ and output parameters $(\overline{T}_e, \phi_e)$ [1,2], is:

$$\overline{T_e}(p) = A_s \overline{T}_e(p) + B_s \phi_e(p)$$
$$\overline{\phi_0}(p) = C_s \overline{T}_e(p) + D_s \phi_e(p)$$

Where $\overline{T_0}$ and $\overline{T_e}$ (resp. ϕ_0 and ϕ_e) are temperature (resp. flux) Laplace transform at $x = 0$ and $x = e$.

$\begin{pmatrix} A_S & B_S \\ C_S & D_S \end{pmatrix}$ is the system transfer matrix given by:

$$\begin{pmatrix} A_S & B_S \\ C_S & D_S \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ h_o S & 1 \end{pmatrix} \begin{pmatrix} A_1 & B_1 \\ C_1 & D_1 \end{pmatrix} \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} A_1 & B_1 \\ C_1 & D_1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ h_e S & 1 \end{pmatrix}$$

$\begin{pmatrix} A & B \\ C & D \end{pmatrix}$ is the granular medium quadruple

$$A = D = \cosh(k \times e), B = \frac{sinh(k \times e)}{\lambda k S} \text{ and } C = \frac{\lambda k S}{sinh(k \times e)}$$

with $k = \sqrt{\frac{p}{a}}$ and $AD - BC = 1$ (passive system)

$\begin{pmatrix} 1 & 0 \\ h_0 S & 1 \end{pmatrix}$ and $\begin{pmatrix} 1 & 0 \\ h_e S & 1 \end{pmatrix}$ are thermal losses quadruples of the irradiated face and the non irradiated face respectively, and $\begin{pmatrix} A_1 & B_1 \\ C_1 & D_1 \end{pmatrix}$ is the metallic plates quadruple.

With $A_1 = D_1, B_1 = 0$ and $C_1 = p \times \rho_1 \times c_1 \times e_1 \times S$

$\rho_1 \times c_1$: metallic plates volume heat. Retained values are given in [3]:
Copper: 3,397 kJ/m^3.K
Aluminium: 2,322 kJ/m^3.K
e_1 (thickness plate) : 1mm

The temperature response $\overline{T}e(p)$ relative to an impulsion $\phi_0(p)$ is given by:

$$\overline{T_e}(p) = \frac{\phi_0(p)}{Cs}$$

where:

$$Cs = \frac{\lambda S}{e}(R(p) + I(p) + RI(p) + K(p))$$

Analytical expressions of $R(p),\ I(p),\ RI(p)\ et\ K(p)$ are given in annex.

2. TEMPERATURE EVOLUTION ON THE NON IRRADIATED FACE

Output temperature Laplace transform (non irradiated face) $\overline{T}e(p)$ is a function of both $\phi_0(p)$ and Cs. $\phi_0(p)$ could be modelled, according to the thermal impulsion imposed by the experimental device, as follows

$$\phi_o(p) = \int_o^{t_o} \left(\frac{q}{t_o}\right) e^{-pt_o} dt = \frac{q \times (1 - e^{-pt_o})}{p \times t_o}$$

q is maintained constant during the lighting period t_o.

R(p) explicit losses at input and output faces through the Biot numbers $Bi_0 = \frac{h_0 e}{\lambda}$ and $Bi_e = \frac{h_e e}{\lambda}$.

To extend the model to consolidated materials, we only have to eliminate terms taking into account plates thermo-physical characteristics, *I(p)*, as well as interactions, *RI(p)*. The temperature response could be rewritten as:

$$\overline{T_e}(p) = \left(\frac{e}{\lambda S}\right) \frac{\phi_0(p)}{(R(p) + K(p))}$$

For the particular case of an ideally insulated material with a Dirac impulsion type ($\phi_0(p)$ = q when $t_o \to 0$), the output face temperature response could be given by:

$$\overline{T}_e(p) = \left(\frac{e}{\lambda S}\right)\frac{\phi_0(p)}{K(p)}$$

Which coincide with the PARKER model [4]. To calculate the temperature of the non irradiated face $T_C(e,t)$, we've used numerical inverse method [5].

3. MEASUREMENT METHOD

A short thermal impulsion is applied to one sample's face insulated laterally from the surrounding. The other sample's face temperature measurement enables one to deduce the apparent thermal diffusivity. In fact, the corresponding experimental thermogram could be exploited using existing theoretical models [1,2]. The thermal diffusivity measurement device is constituted of Box B, Fig.2, with internal reflecting faces. The bottom face presents an opening through which six glow lamp emits 600 to 1000 Watts to the sample placed between box B and capacity A. For the granular medium we use a slab of dimension 27 × 27 × e cm. The two principal faces, the irradiated and the non irradiated one are composed of metallic plates as could be copper and aluminium, of thickness e_1. The sample is then assimilated to a sandwich metal-material-metal.

4. THERMAL DIFFUSIVITY IDENTIFICATION

The calculated temperature response $T_C(e,t)$ obtained using inverse numerical method for $\overline{T}_e(p)$ depends on three parameters, namely, h_0 , h_e , and a and could be written as

$$T_C(e,t) = T_C(e, h_O, h_e, a, t)$$

These three parameters are estimated from both the analytical expression issued from the model $T_C(e,t)$ and experimental result $T_{\exp}(e,t)$ through minimising J defined as

$$J(a) = \sum_1^n (T_{\exp}(e,t) - T_C(e, h_O, h_e, a, t))^2$$

Where n is the number of the chosen experimental points. To make easy this task, we've reduced the parameters number of criteria J. The procedure consists in determining the ratio $\frac{h_0}{h_e}$ minimising the relative error made when measuring the thermal diffusivity, a, (Table 1) taking into account the experimental conditions and the values of h_0 and h_e given in [6].

The minimisation criteria of J(a)

$$J(a)=\sum_{1}^{n}(T_{\exp}(e,t)-T_{C}(e,a,t))^{2}$$

is obtained using an iterative method [2].

$\frac{h_0}{h_e}(\%)$	1	10	20	30	40
$\frac{\Delta a}{a}(\%)$	6,614	3,079	2,952	2,75	2,74

Table 1. Influence of thermal losses on the thermal diffusivity measurement

5. RESULTS AND COMPARISON

5.1 Granular medium

The granular medium is constituted of spherical glass-ball expanded of air. Table 2. gives results of the thermal diffusivity according to both, the counting of the experimental measurement obtained at the non irradiated face (see annexe), and the identification method for different thermal impulsion period. We can see dispersion in results: relative difference is of 15% when comparing with Degiovanni counting method [7], and of about 5 % when comparing with Yezou counting method [8]. This difference traduce the error caused when using metallic plates in the measurement of thermal diffusivity of powders medium.

	Thermal diffusivity [7]			Thermal diffusivity [8]		Identification method
Irradiation period t_0 (s)	$a_{2/3}$	$a_{1/2}$	$a_{1/3}$	$a_{5/6}$	$a_{1/2}$	J(a) criteria
10	1,38	1,32	1,47	1,36	1,37	1,34
15	1,16	1,25	1,35	1,28	1,28	1,27
20	1,23	1,36	1,42	1,40	1 40	1,41
25	1,16	1,30	1,35	1,34	1,35	1,34
30	1,24	1,30	1,23	1,33	1,33	1,32

Table 2.a: Apparent thermal diffusivity $(a\times10^{7}\,m^{2}/s)$
Comparison: Counting- Identification

We can also see that the support plate choice (different thermophysical properties) does not have sensitive influence on thermal diffusivity measurement of the considered porous medium (Table 2. b)

	Thermal diffusivity [7]			Thermal diffusivity [8]		Identification method
Support Plates	$a_{2/3}$	$a_{1/2}$	$a_{1/3}$	$a_{5/6}$	$a_{1/2}$	Critère J(a)
Copper	1,23	1,36	1,42	1,40	1,40	1,41
Aluminium	1,30	1,41	1,45	1,44	1,45	1,40

Table 2.b: Apparent thermal diffusivity $(a \times 10^7 m^2/s)$
Support Plates Influence

5.2 Compact medium

We consider in this study air expanded polystyrene based concrete of 60 days (sample significance in annex). Table 3 allows for the comparison between thermal diffusivity measurement relative to 4 slabs using identification and counting methods. Irradiation period t_0 is fixed to 30 seconds. Obtained results from identification method and Yezou counting method are in good agreement. In fact, these two methods have been suggested for a large dimension construction material characterisation; whereas the difference between the three values of a issued from the Degiovanni counting method, is more important.

	Thermal diffusivity [7]			Thermal diffusivity [8]		Identification method
Sample	$a_{2/3}$	$a_{1/2}$	$a_{1/3}$	$a_{5/6}$	$a_{1/2}$	J(a) criteria
P6/18/1572	4,12	3,96	4,13	3,98	3,95	4,01
P6/28/1371	3,73	3,76	3,80	3,78	3,74	3,73
P6/35/1235	3,36	3,25	3,39	3,26	3,23	3,26
P6/72/542	2,32	2,49	2,51	2,50	2,48	2,49

Table 3: Apparent thermal diffusivity $(a \times 10^7 m^2/s)$
Polystyrene based concrete. Comparison: Counting- Identification

6. CONCLUSION

The described method allows a good interpretation of the apparent thermal diffusivity measure experiment which, uses a short period impulsion (of about 20 seconds). The method presents the advantage in using, when necessary, all the experiment points, attributing to them a weight proportional to their precision and hence more accuracy. It allows also the evaluation of perturbation that affect the thermal diffusivity measurement of granular medium when the sample is presented as a ’’ Sandwich ’’ metal-granular medium-metal. And finally, as a methodology, the followed procedure is interesting because it’s easily adapted for thermal diffusivity measurement of a compact material.

REFERENCES

[1] DEGIOVANNI A.
Thermal conduction in multilayer slab with internal sources: using a quadruple method, Int. J. Heat and Mass Transfer. Vol. 31, n°3, pp. 553-557, (1988)

[2] El BOUARDI A., EZBAKHE H., MENGUY G.,. BURET J.B
Influence of water content and volumic mass on the thermophysical characteristics of granular materials used in insulation. World Renewable Energy congress, Reading, UK, pp. 2743-2748, 23-28, September, (1990)

[3] SACADURA J. F
Initiation aux transferts thermiques. Technique et documentation. (1979)

[4] PARKER W.J., JENKINS R.J., BUTTLER G.P., ARBOTT G.L.
Flash method of determining thermal diffusivity, heat capacity and thermal conductivity, J. Appl. Phys., 32, n°9, (1961)

[5] GRAVER-STEHFEST, WOO
Algorithm 138. Numerical inversion of Laplace transforms. A.C.M. 13, 47-49 (1970)

[6] AIDE MEMOIRE DU THERMICIEN
Aide mémore de du Thermicien. Edition Européenne Thermique et Industrie. E.E.T.I, (1987)

[7] DEGIOVANNI. A
Diffusivité et méthode flash, Revue Générale de Thermique, n°185, pp :420-442. (1977)

[8] YEZOU R.
Contribution à l’étude des propriétés thermophysiques des matériaux de construction cohérents et non cohérents. Thèse de Docteur- Ingénieur, INSA, Lyon, France, (1978)

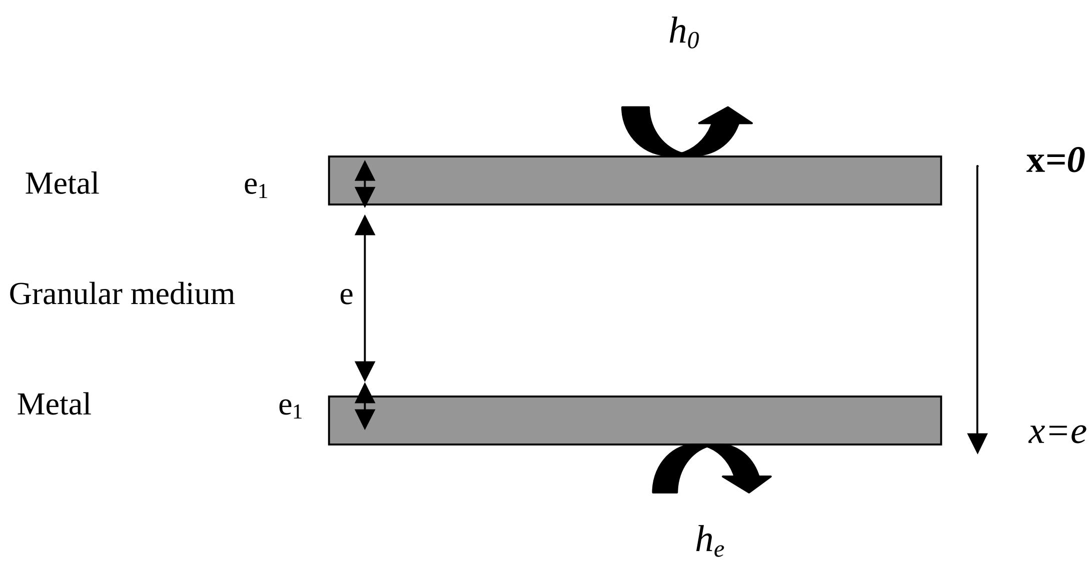

Figure 1.a : A sample slab between two metallic plates with losses on the two faces

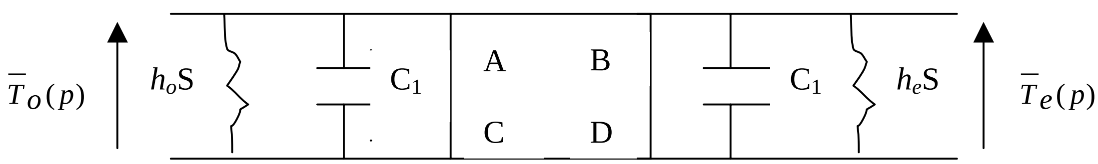

Figure 1.b: Equivalent electrical scheme

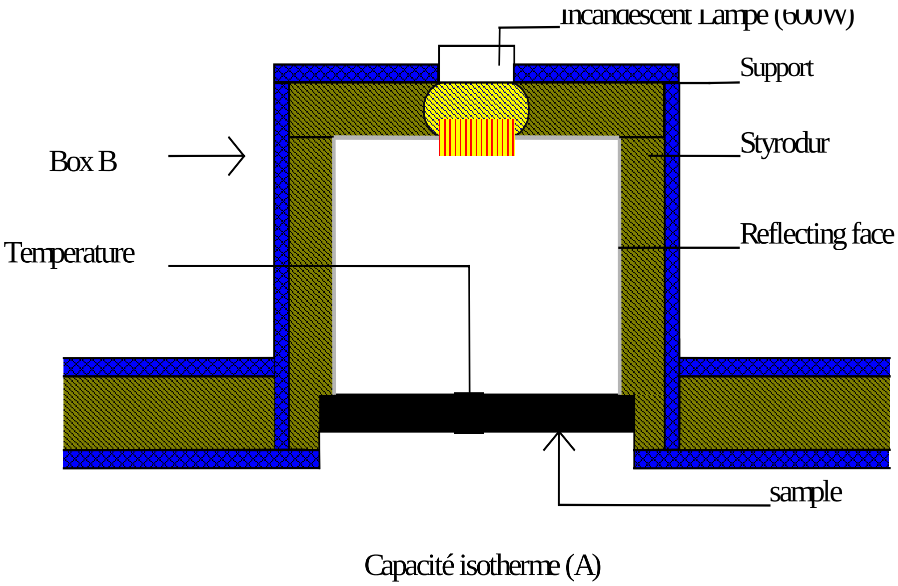

Figure 2. Box for thermal diffusivity measurement

ANNEXE

Degiovanni counting method[7]:

$$a_{2/3} = (e/t_{5/6})^2 (1{,}15 \quad t_{5/6} - 1{,}25 \quad t_{2/3})$$

$$a_{1/2} = (e/t_{5/6})^2 (0{,}761 \quad t_{5/6} - 0{,}926 \quad t_{1/2})$$

$$a_{1/3} = (e/t_{5/6})^2 (0{,}67 \quad t_{5/6} - 0{,}862 \quad t_{1/3})$$

Yezou counting method [8]:

$$a_{5/6} = (e^2 / t_{5/6} + t_{0/2})\ (0{,}713\ A^2 - 1{,}812 A + 1{,}037)$$

$$a_{1/2} = (e^2 / t_{1/2} + t_{0/2})\ (-0{,}4032\ A^2 + 0{,}1103 A + 0{,}2027)$$

$$A = (t_{1/2} + t_{0/2}) / (t_{5/6} + t_{0/2})$$

ti/j is time corresponding to i/j of maximum temperature in non exciting face.

R(p), I(p), RI(p) and K(p) expressions

$$R(p) = (B_{io} + B_{ie}) ch(k/p)) + B_{io}.B_{ie}\ ch(k/p))/k(p)$$

$$I(p) = (p.\rho_1 c_1 e_1 \quad e/\lambda)^2 sh(k(p))/k(p) + 2p.\rho_1 c_1 e_1\ e/\lambda.ch(k/p))$$

$$RI(p) = (p.\rho_1 c_1 e_{11} e/\lambda)(B_{i0} + B_{ie}) sh(k/p))/k(p)$$

$$K(p) = k(p).sh(k/p))$$

With: $k(p) = (p.e^2/a)^{1/2}$

code of Sample measurement

PDm/θ/ρ$_o$

P: Polystyren concrete

Dm: balls average diameter

θ: Volumic fraction

ρ_o : density at 28 days